Mixed Correlators in 3D Conformal Field Theories

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Outline

Background on CFT and the Conformal Bootstrap

Prior Results in the Space of 3D Ising-Like CFTs

New Results from 3-Point Symmetry

New Results from “Theta Scan”

Introduction to CFT

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- CFT plays a central role in the development of modern physics, appearing in the study of phase transitions, string theory, and dualities in quantum gravity.
- A CFT gives rise to **correlation functions**:

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\dots\mathcal{O}_n(x_n)\rangle \, .$$

Conversely, these correlators give information about a CFT.

Correlation Functions

- Conformal symmetry constrains the form of all two point correlators
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$C_{\mathcal{O}}$ and C_{123} are constants depending only on the field $\mathcal{O}$.
$\Delta_{\mathcal{O}}$ is called the **scaling dimension** of $\mathcal{O}$.

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- This constraint, together with the OPE allows us to determine whether a given set of CFT data (field scaling dimensions, etc.) can give rise to an actual CFT.

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- Key Question:

Are there other 3D "Ising-like" CFTs?

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  Literally none.

Method of Attack

- Expansions of functions corresponding to the associativity of the correlators $\langle\sigma\sigma\sigma\sigma\rangle$, $\langle\varepsilon\varepsilon\varepsilon\varepsilon\rangle$, $\langle\sigma\sigma\varepsilon\varepsilon\rangle$ give us the constraints for the CFT.

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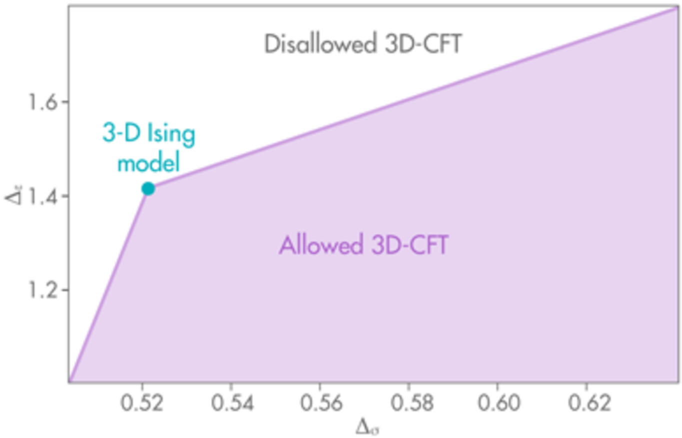
- Essentially, we want to see whether a given series can equal zero
- This leads a task in semidefinite programming, implemented by David Simmons-Duffin's 'SDPB'

Prior Results

Using just the associativity conditions on $\langle\sigma\sigma\sigma\sigma\rangle$[1]:



[1] No assumption on the number of relevant operators

Prior Results

Quantamagazine | Physics | Mathematics | Biology | Computer Science | All Articles

THEORETICAL PHYSICS

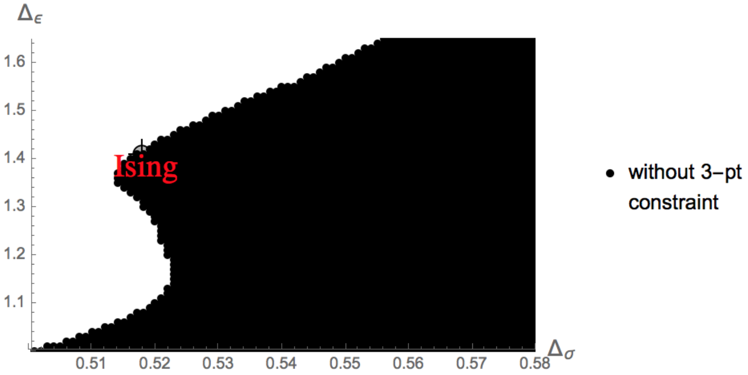
Physicists Uncover Geometric 'Theory Space'

16

A decades-old method called the "bootstrap" is enabling new discoveries about the geometry underlying all quantum theories.

Prior Results

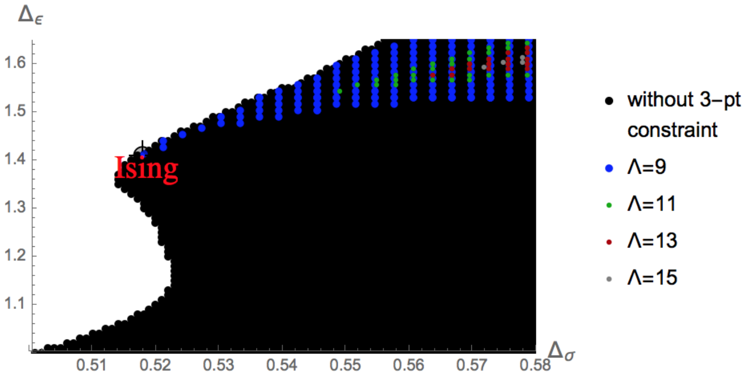
Using mixed correlators[2]:



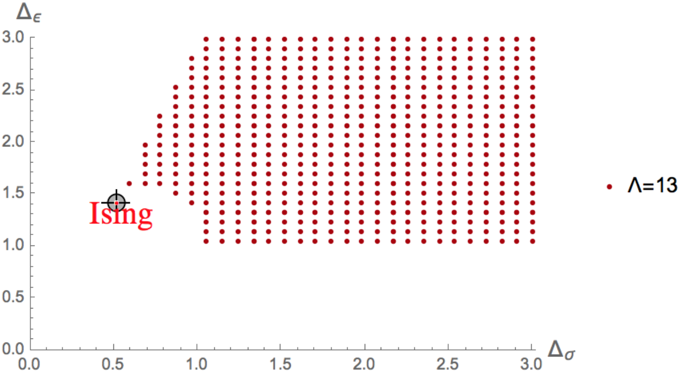
[2] Plotted from Aaron Hillman's work last semester

Enforcing 3 point Symmetry

Using the package 'cboot' that makes use of a further symmetry in the 3-point coefficients $\lambda_{\sigma\epsilon\sigma} = \lambda_{\sigma\sigma\epsilon}$:



Not very close to being global...



Scanning over 3-point Functions

Define $\tan\theta = \lambda_{\sigma\sigma\epsilon}/\lambda_{\epsilon\epsilon\epsilon}$.
We now scan over all possible $(\Delta_\sigma, \Delta_\epsilon, \theta)$:

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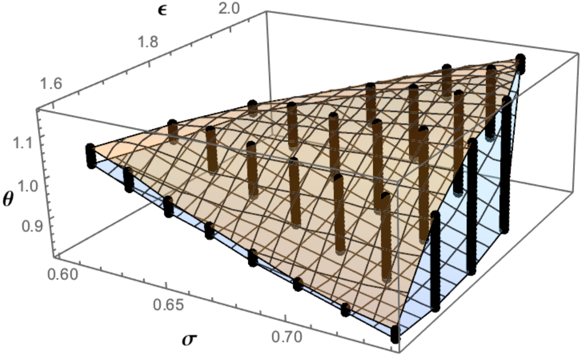

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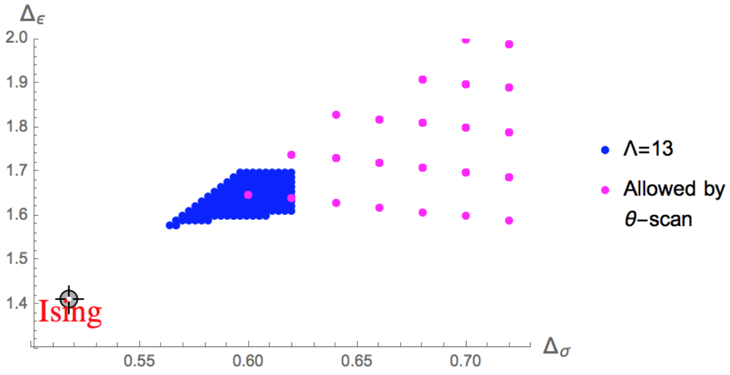
Projection to 2D:



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Next steps:

1. See how we can get results using much higher Λ, e.g. $\Lambda = 30$.
2. Trace the size of the region excluded as a function of $\Lambda \to \infty$